good agreement with the experimentally observed current pulses shown in Fig. 2. This good agreement serves to demonstrate the validity of the model.

It will be convenient to analyze the data with the value of current at $t=0$ called i_i, and the current at shock-wave transit time t_0, called i_f. From Eq. (22) we see that

$$i_f/i_i = \alpha^2(1-u/U)^{-2}. \tag{23}$$

Thus, this ratio of currents provides an experimental measure of the ratio of the strained-to-unstrained permittivity. The current value when the shock enters the disk provides a direct convenient measure of the piezoelectric polarization. From Eq. (22),

$$P_\eta = \frac{t_0\alpha}{A(1-u/U)}\, i_i. \tag{24}$$

The direct relation between current and piezoelectric polarization shown in Eq. (24) allows each current measurement at a particular strain to serve as a direct measure of the polarization. The electrostatic model leading to Eq. (24) does not include the effect of electromechanical coupling on the current wave shape; however, a small correction to the basic model will permit accurate solutions which incorporate electromechanical coupling.

D. Electromechanical Coupling

Several authors have obtained analytical solutions for the effect of electromechanical coupling on a short-circuited current from piezoelectric disks.[32-34] The total correction to the polarization computed from the electrostatic model is only about 1% for the present experiments; however, the accuracy of the present experiments makes this correction necessary. Stuetzer[32] has shown that electromechanical coupling has the effect of modifying the step-function current pulse in such a way that $i(t) = i_i e^{Bk^2 t/t_0}$, where k^2 is the electromechanical coupling, $e_{11}^2/\epsilon_1^s c_{11}^D = 0.0084$,[21] and B is a constant which depends upon the acoustical boundary conditions at the electrodes. Values for B are 1.0, 1.5, or 2.0 depending upon whether the acoustic impedances of the two electrodes are, respectively, matched-matched, matched-free, or free-free.

For the present experiments the configuration is acoustically matched at the input electrode and has Epoxy potting with an acoustic impedance one-fifth the value of X-cut quartz on the opposite face. In addition to the Epoxy, a solder joint and wire connect to the electrode remote from the impact face. Linear interpolation with the acoustic impedance shows that the Epoxy would cause a value of $B=1.4$. Our low-strain experiment in which the current-time response is measured provides a direct measure of $i(t)$ which is in good agreement with $B=1.4$. These considerations lead to the result that the ratio of currents at small strains R caused by electromechanical coupling is $R=e^{1.4k^2}$. A small increase in R with strain is anticipated from the preliminary data analysis which determines values for e_{11}, ϵ_{11}, and c_{11} at large strains. From these results it follows that

$$R(\gamma) = 1.012 + 0.28\gamma. \tag{25}$$

The initial current-jump value i_i used to compute the piezoelectric polarization is unaffected by electromechanical coupling. However, the experimentally observed ratio of final current to initial current, i_f/i_i, includes a contribution due to the electromechanical coupling. Thus, the value of α corresponding to the ratio of the permittivity of the strained-to-unstrained material is less than that predicted in the electrostatic model without electromechanical coupling. The computation of a value for α which includes correction for electromechanical coupling is then

$$\alpha = (1-u/U)[1+(i_f/i_i)-R]^{1/2}. \tag{26}$$

The electrostatic analysis with minor modification for electromechanical coupling is used to analyze each current-pulse measurement. The piezoelectric pulse analysis indicates that piezoelectric polarization may be computed from the value of the current jump i_i with the aid of Eq. (24). The value of α used in Eq. (24) is obtained from a best fit to the values of α computed from Eq. (26).

V. RESULTS

Experiments were conducted over a large range of strain $(2.4\times10^{-3}$–$7\times10^{-2})$ to determine the limit of elastic response. Experiments at strains from 4.3×10^{-2} to 7×10^{-2} were observed to show current-time wave shapes with highly nonlinear behavior which became increasingly pronounced with increasing strain. The distorted wave shapes indicate that the model proposed in Sec. IV is insufficient to determine the piezoelectric response for strains greater than 4.3×10^{-2}. The current pulses are consistent with that expected from inelastic response[35] and serve to show that the limit of elastic response in shock-loaded X-cut quartz is 4.3×10^{-2}. Accordingly, the results to be shown are limited to the experiments at strains less than this elastic limit.

To determine the piezoelectric polarization from the observed current pulses, the shock velocity vs input particle velocity and the current-ratio data were represented by statistical fits. To improve the statistical confidence in the result, wave-velocity data from the previous study[11] for strains between 2.5×10^{-2} and 4.3×10^{-2} are included with measurements from the present investigation. A linear fit to the data gives the result

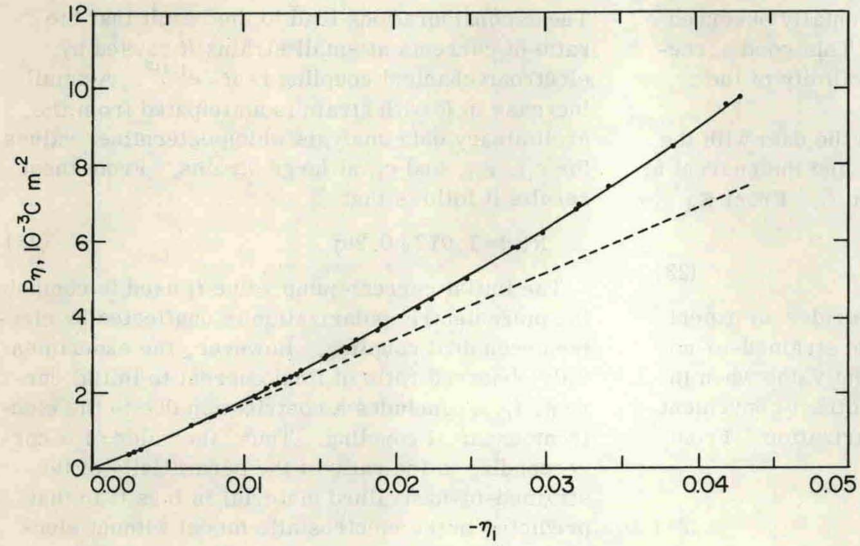

FIG. 4. Piezoelectric polarization P_η observed at various compressive strains. The dashed line is plotted to illustrate the linear response which would be observed if the direct nonlinear constant were zero. The line through the data is a plot of the quadratic fit which is observed to give an excellent representation to the data. The difference between the observed polarization and the dashed line serves to illustrate the contribution of the nonlinear constant at the various strains.

$$U(u) = [(5.724 \pm 0.018) + (0.312 \pm 0.112)u] \times 10^3 \text{ m sec}^{-1}, \quad (27)$$

where the ± indicates the standard error and $u_{max} = 250$ m sec^{-1}. The experimental error of ±0.5% is felt to be the major contributor to the standard error.

Even though the wave-velocity measurements are not as precise as would be desirable, it should be observed that the piezoelectric-constant values determined in the present configuration are independent of the wave velocity. This result is readily obtained by observing that the polarization, Eq. (24), and strain, Eq. (13), are both inversely proportional to the velocity; hence, the ratio of polarization to strain is independent of wave velocity.

The experimentally observed ratios of final current to initial current are represented by a linear least-squares fit which expresses the value for the strain dependence of α as

$$\alpha = 0.9996 + (0.463 \pm 0.130)\gamma. \quad (28)$$

Unfortunately, for strains greater than 3×10^{-2} the current-vs-time pulses showed evidence of shock-induced conductivity. The conductivity was relatively small and did not affect the first current values i_i from which the polarization was computed. However, final current values i_f were sufficiently changed by the conductivity such that values of α could not be calculated from the experimental data in this strain range. The value of α is not likely to undergo an abrupt change; hence, the values for α determined up to 3×10^{-2} are linearly extrapolated to strains up to 4.3×10^{-2} to interpret the large-strain data.

Piezoelectric-polarization values were calculated with the aid of Eqs. (24), (27), and (28) for each of 35 measured current pulses. The resulting piezoelectric-polarization values are plotted against the quadratic strain in Fig. 4.

To aid in interpretation of the nonlinear effects, a line with slope equal to the linear constant is shown. The difference between the linear contribution and the observed polarization is that due to the nonlinear contribution. It can be readily observed that the experiments utilize strain for which the nonlinear contribution is negligible as well as strains for which the nonlinear contribution is pronounced.

The line drawn through the data is a quadratic least-squares fit. It is apparent that the quadratic fit gives an excellent representation to the data. This observation confirms that the form of the piezoelectric constitutive relation expressed in Eq. (10) provides an excellent quantitative representation to the response of X-cut quartz to strains as large as 4.3×10^{-2}. The data further demonstrate that contributions of higher-order terms are negligibly small $(\partial^2 e_{11}/\partial\eta_1^2 < \sim 10^{-1}\text{ C m}^{-2})$.

The quadratic fit to the piezoelectric-polarization data yields the result

$$P_\eta = (0.1711 \pm 0.00094)\eta_1 + (1.32 \pm 0.024)\eta_1^2, \quad (29)$$

where the units are C m^{-2}. The linear term is the value of the e_{11} piezoelectric stress constant, and the second term is a value for $\frac{1}{2}(\partial e_{11}/\partial\eta_1)$. The measurements provide a value for the linear constant with a standard error of 0.55%, the most accurate value achieved to date for this constant. The nonlinear constant is obtained with a standard error of 1.9%. Since the standard errors are consistent with the experimental errors, the material exhibited sample-to-sample variations which were significantly less than the experimental error.

A quadratic fit to the data for strains less than 3×10^{-2}, where conductivity was not observed,